

## Fatigue life prediction of multiple site damage based on probabilistic equivalent initial flaw model

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**Abstract.** The loss of strength in a structure as a result of cyclic loads over a period of life time is an important phenomenon for the life-cycle analysis. Service loads are accentuated at the areas of stress concentration, mainly at the connection of components. Structural components unavoidably are affected by defects such as surface scratches, surface roughness and weld defects of random sizes, which usually occur during the manufacturing and handling process. These defects are shown to have an important effect on the fatigue life of the structural components by promoting crack initiation sites. The value of equivalent initial flaw size (EIFS) is calculated by using the back extrapolation technique and the Paris law of fatigue crack growth from results of fatigue tests. We try to analyze the effect of EIFS distribution in a multiple site damage (MSD) specimen by using the extended finite element method (XFEM). For the analysis, fatigue tests were conducted on the centrally-cracked specimens and MSD specimens.

**Keywords:** fatigue life prediction; equivalent initial flaw size; multiple site damage; back extrapolation; extended finite element method.

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### 1. Introduction

The fatigue crack growth (FCG) process is stochastic in nature. Fatigue life prediction is critical for the design and maintenance planning of many structural components, but is still challenging despite extensive researches over the past several decades (Yongming and Sankaran 2009, Dawn *et al.* 2011). One problem in the fracture mechanics-based life prediction which is the physical basis for damage tolerance analysis is to determine the initial crack size for crack growth analysis. One practice is to use an empirically assumed crack length such as 0.25-1 mm for metals (Gallagher 1984, JSSG-2006 1998, Karen *et al.* 2004, Merati and Eastaugh 2007). An alternative way is to use results from nondestructive inspection (NDI) (Krasnowski *et al.* 1991). However, due to limitations of NDI methods and the measurement of initial crack size, the initial flaw size (IFS) is below the current detection technology of the NDI. If the NDI detection limit is chosen as the initial flaw size,

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it will result in a very conservative design (Forth *et al.* 2002).

Structural components unavoidably are affected by defects such as surface scratches, surface roughness or weld defects of random sizes, which usually occur during the manufacturing and handling process (Artley 1989). Therefore, the initial flaw of the material can be considered as a material property. These defects are shown to have an important effect on the fatigue life of the structural components (Tong 2001, Kim *et al.* 2008).

Two concepts have been developed as useful design tools to make life predictions for aircraft structural reliability problems. They are the equivalent initial flaw size (EIFS) distribution and the distribution of time-to-crack initiation (TTCI). The EIFS is an artificial crack size, which is derived from the distribution of fatigue cracks occurring later on during the service life. The period of crack initiation or TTCI is defined as the time in cycles or flight hours, which takes for a non detectable crack from the beginning of fatigue loading to grow to a reference crack size. The calculation of EIFS is usually performed using a back-extrapolation method. Yang and Manning (1980) used this back-extrapolation technique to obtain the EIFS distribution of Al 2024-T351. White *et al.* (2005) used a probabilistic fracture approach to derive the equivalent pre-crack size (EPS), which is also based on the back-extrapolation method. Molent *et al.* (2006) used a back projection of the experimental crack growth curve to time zero to derive the EPS for Al 7050. The major problem using the back-extrapolation method is that the obtained EIFS seems to be dependent on stress level (Moreira *et al.* 2005). It is desirable to view EIFS as a material property indicating the initial quality of the material and not connected to the applied stress level; this would make the EIFS applicable to a wide range of stress levels.

It should be noted that EIFS is not a physical quantity. It is a quantity extrapolated from experimental data simply to facilitate life prediction by using only long crack growth analysis and avoiding the difficulties of short crack growth modeling because the mechanism of small-crack growth has not been fully understood. The other difficulty is that small-crack growth strongly depends on the material microstructure and has very large uncertainties caused by the randomness of grain size, grain orientation and initial flaw shape (Yongming and Sankaran 2009).

Another important issue in the fracture mechanics-based life prediction is the stochastic nature of the applied load (Amanullah *et al.* 2002). It is well known that the crack growth mechanism is influenced by the stress sequence and interaction associated with the arrangement of the load spectra (Matej *et al.* 2009). Given that crack growth is driven by stress intensity factor (SIF), the initial flaw size is important to affect crack growth because SIF increases as flaw size increases.

The main problems of aeronautical structures are damages due to fatigue, a phenomenon accentuated in areas of stress concentration, as for example the connections of fuselage panels, often done by riveting. The connection of components has the widespread fatigue damage (WFD) consisting of multiple fatigue cracks. One of WFD forms is the multiple site damage (MSD) in which a major crack interacts with other cracks in various locations. When a critical situation is reached, cracks link up to form a large crack which abruptly reduces the residual strength of the damaged structural member. Therefore, it is important to estimate the fatigue life before the crack link-up under the MSD. MSD problems are analyzed typically by using numerical methods rather than theoretical ones because of the complicated interactions between multiple cracks. The use of numerical methods is still limited by the change of the domain geometry due to the simultaneous growth of multiple cracks. With traditional methods, it is necessary remeshing the computational domain as MSD cracks grow. The method in our study for MSD problems is based on the extended finite element method (Belytschko and Black 1999, Moes *et al.* 1999) which has been extended to

many applications; crack growth with friction (Dolbow *et al.* 2000), arbitrary branched and intersecting cracks (Daux *et al.* 2000), three-dimensional crack propagation (Sukumar *et al.* 2000), material discontinuity problems (Belytschko *et al.* 2001), cohesive crack models (Zi and Belytschko 2003), dynamic fracture problems (Belytschko *et al.* 2003), etc. We show here how the method can be used to model the growth of multiple cracks and the junction of cracks. The benefit of using the extended finite element method is that one can model the discontinuities without remeshing.

In this study, the value of EIFS is calculated by using the back extrapolation technique and the Paris law of fatigue crack growth. The fatigue test on several rivet specimens made of Al 2024-T3 was conducted. We analyzed the problem of an example considering the effect of EIFS distribution on MSD specimen. An example problem was calculated by the extended finite element method (XFEM) to analyze MSD problems.

## 2. Initial flaw size distribution

In order to make reliable predictions, data regarding the initial flaw qualities must be known. However, available NDT methods at present cannot provide adequate information concerning statistical distributions of initial flaws. In addition, it has been found that not all of initial flaws, such as notches or scratches present on the surface of the components representing crack initiation flaws. As a result, EIFS and TTCI have been developed as a useful design tool to make life predictions for aircraft structural reliability problems.

### 2.1 Equivalent Initial flaw size (EIFS) distribution

The nucleation phase has a strong influence on the fatigue life of structures. Several random crack propagation studies identify the initial crack size as one of the most important factors affecting crack growth uncertainty. However, random loading introduces additional uncertainty in fatigue life. Crack growth in structures depends on the amplitude, stress ratio, and frequency of the load. Due to the random nature of variable loading, it is difficult to model all these influential parameters correctly. Overloads are known to retard crack growth while underloads accelerate crack growth relative to the background rate (Huang *et al.* 2007). These interactions, which are highly dependent upon the loading sequence, make the prediction of fatigue life under variable amplitude loading more different and complex than under constant amplitude loading.

Initial flaws of a structure are not detectable. Furthermore, not all flaws are propagated from an initial defect of the same size. For this reason, the equivalent initial flaw size concept was introduced by Rudd and Gray (1978) and developed by Yang and Manning (1980) as an analysis technique to represent the initial quality of structural details in the durability analysis.

Initial fatigue quality of a durability critical component is often characterized by EIFS. As a starting crack length for a crack growth calculation, the EIFS is a fictitious crack length which yields an equivalent fatigue life that can be found in the conventional fatigue test.

The distribution of fatigue cracks at a particular time may be difficult and costly to determine. This kind of information usually requires a tear down inspection, possibly a full-scale fatigue test or retired airframes. Fatigue cracks detected during in-service inspections of structural components or the ones obtained from coupon testing in laboratory may also serve as a starting point to develop the EIFS distribution.

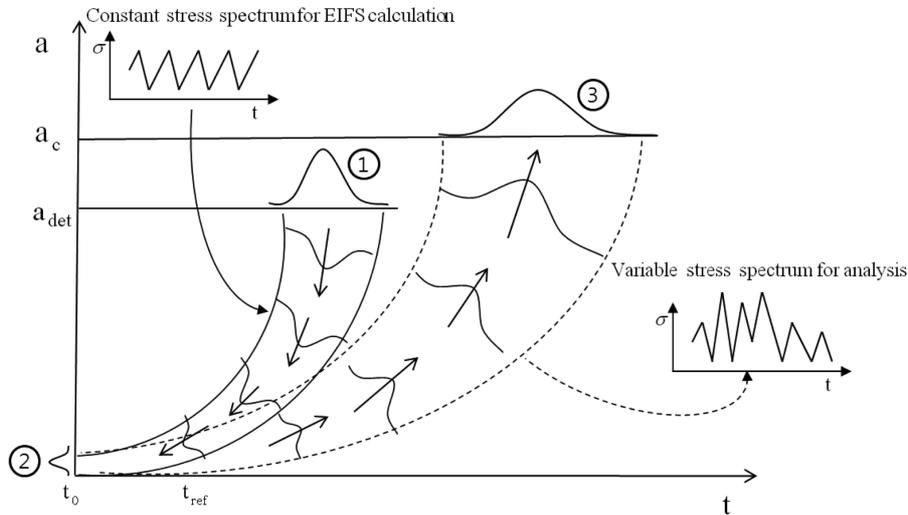


Fig. 1 Utilization of EIFS distribution for different variable stress spectrum

In general, the EIFS depends on the test variables, fractographic results, and crack size range over which the fractography has been carried out, on the crack growth model used for the back extrapolation (Moreira *et al.* 2007). In practice, the EIFS distribution is used to predict a distribution of crack growth lives for a given structure and loading condition similar to that from which the EIFS distribution was derived.

The fracture mechanics-based life prediction is required to calculate the fatigue life for different variable stress spectrum. Given that crack growth is driven by stress intensity factor (SIF) of stress spectrum, the initial flaw size is important to affect crack growth because SIF increases as flaw size increases. Therefore, the EIFS distribution can be utilized after the fatigue life distribution (① in Fig. 1) at the detectable crack length ( $a_{det}$ ) is obtained from the experiment result under constant stress spectrum. the EIFS distribution (② in Fig. 1) is calculated from the fatigue life distribution (① in Fig. 1) at the detectable crack length ( $a_{det}$ ) under constant stress spectrum by using the Paris equation and extrapolation technique. In case of different variable stress spectrum applied, the fatigue life distribution (③ in Fig. 1) at failure ( $a_c$ ) can be predicted from the obtained EIFS distribution (② in Fig. 1).

## 2.2 Back extrapolation technique by using paris law

The distribution of EIFS can be determined by back-extrapolating distribution of fatigue cracks according to a master crack growth function to zero time or a reference position time serving to represent the initial time of the assessment. The EIFS will play a role of basis in time when it is grown forward.

The fatigue life of structural or mechanical members is defined as the number of loading cycles until the member fails due to crack growth. The fatigue life can be divided into three main periods: crack initiation, crack growth, and finally, overload failure. Here we are interested in the crack growth period. The relation between crack growth and stress intensity factor in the period of crack growth is almost linear in a log-log plot. A Paris law raised to higher power of stress intensity

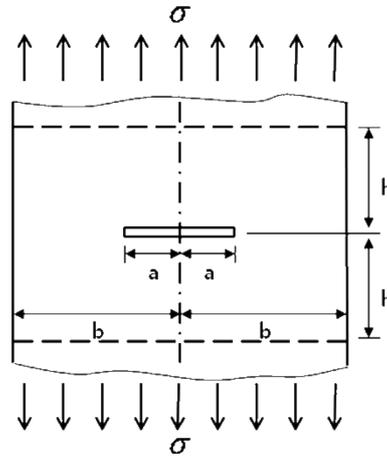


Fig. 2 Type and load condition of the centrally-cracked specimen

factor is directly proportional to the fatigue crack growth rate as follows.

$$\frac{da}{dN} = C(\Delta K)^m \tag{1}$$

$da/dN$  is the fatigue crack growth rate,  $\Delta K$  is the stress intensity factor ( $K_{max} - K_{min}$ ) and  $C, m$  stand for  $f$  (material variables, environment, frequency, temperature, stress ratio).

The EIFS values were obtained for a finite plate with central hole and two symmetrical cracks as shown in Fig. 2. The values of  $m$  and  $C$  used are 3.1 and  $3.0E-8[\text{MPa}\cdot\text{m}^{0.5}]$  (Huang *et al.* 2007). The range of stress intensity factor is possible to be calculated from Tada handbook (Tada *et al.* 2000). The stress intensity factor equation is defined as

$$K_I = \sigma\sqrt{\pi a} \cdot F(a/b) \tag{2}$$

$$F(a/b) = \{1 - 0.025(a/b)^2 + 0.06(a/b)^4\} \sqrt{\sec\frac{\pi a}{2b}} \tag{3}$$

where  $a$  is the half crack length and  $b$  is the half width length in the centrally-cracked specimen.

Eq. (3) is the modification of Feddersen's formula. It can be predicted that the accuracy is 0.1% for any  $a/b$  (Tada *et al.* 2000). The value of EIFS (a) for a symmetric crack was calculated.

### 3. Calculation of equivalent initial flaw size distribution

The EIFS is the size of initial flaw that under-stated cyclic loading would lead to a prescribed endurance. To calculate the value of the EIFS, the back extrapolation technique and fatigue crack growth rules for long-crack regime were used.

#### 3.1 Forward loading by a constant stress to develop detectable flaw distribution

To obtain the distribution of a fatigue life, we conducted fatigue crack growth tests subject to

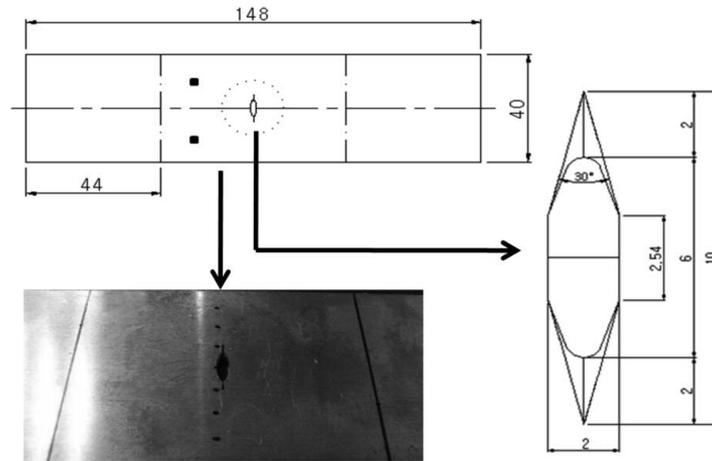


Fig. 3 Details and experiment picture of the centrally-cracked specimen

Table 1 Load conditions and load cycles at failure for the centrally-cracked specimens

Specimen	Frequency (Hz)	$P_{\max}$ (kN)	$P_{\min}$ (kN)	Stress ratio (R)	Cycles at failure
FT-1020	1	11.2	2	0.178	6,350
FT-1030	1	6.7	2	0.297	50,100

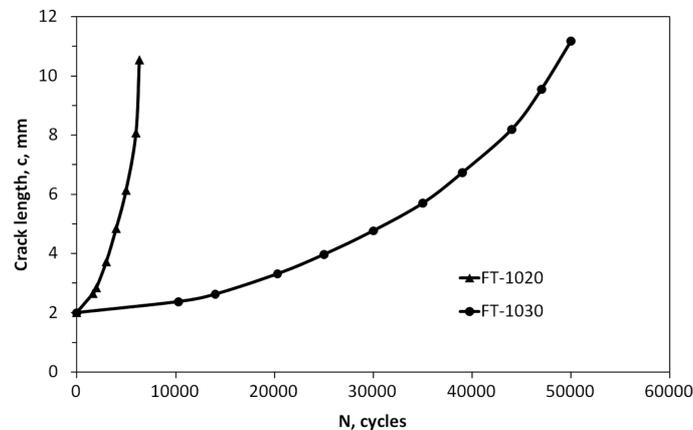


Fig. 4 Behaviors of crack growth for two load conditions in the centrally-cracked specimens

constant amplitude cyclic loading. Specimens are thin sheet plates made of aluminum Al2024-T3. Two centrally-cracked specimens were used following ASTM E 647. Detailed dimensions of the specimens are expressed in Fig. 3. Even though, a crack length of 1 mm at each tip is recommended in ASTM E 647, we had to cut 2 mm at each tip because of the minimum allowable length that can be obtained by laser.

The specimens were tested with a constant amplitude cyclic loading until a rupture occurs. MTS-810 UTM (Universal Testing Machine) was used with a frequency of 1 Hz. The numbers of cycles until a detectable crack was initiated were 1,700 and 10,300 cycles for the specimens FT-1020 and

FT-1030, respectively. Crack length has been measured at different number of cycles to investigate the propagation of the crack. The number of cycles of each specimen at the point of rupture is presented in Table 1. Also, the crack growth length with respect to the number of cycles is presented in Fig. 4.

### 3.2 Back-extrapolation for EIFS inference

The Crack initiation is considered to be one of the two major periods in the fatigue life of a component or structure. The period of crack initiation or TTCI is defined as the time in cycles or flight hours, which takes for a non detectable crack from the beginning of fatigue loading to grow to a reference crack size. The relation between the TTCI distribution and the EIFS distribution can be described as Fig. 5. Yang and Manning (1980) has demonstrated this existence of compatibility between the TTCI distribution function and the EIFS distribution function for the Weibull and the lognormal distributions. A power law matching the crack growth rate is used to consider the crack growth law changing the TTCI distribution function back to the y-axis at zero time to calculate a compatible EIFS distribution.

From the literature, the most common assumption is that the time to failure distribution is lognormally distributed. It was found that the standard deviation for tests on simple unnotched specimens is, in general, greater than that for tests on complex built-up structures of the same material. Table 2 has been taken from reference which expresses the variation of standard deviation with specimen type for aluminum alloys.

The EIFS calculation needs the large amount of test data to develop a reliable EIFS distribution. However, our study hasn't obtained enough data to create a crack distribution at detectable flaw or failure. A distribution of crack at failure can be calculated from assuming lognormal distribution used in a failure distribution. The average in assumed distribution is the fatigue life (6,350 cycles at

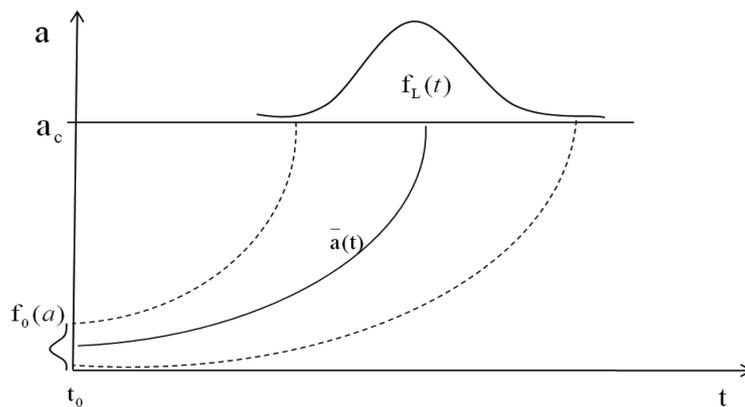


Fig. 5 Process of the compatibility between TTCI and EIFS distribution (Tong 2001)

Table 2 Variation of standard deviation with specimen type for aluminum alloys (Tong 2001)

Type of Specimen	Standard Deviation of Log-Life
Simple, Unnotched Specimens	0.300
Built-up Aircraft Structures	0.176

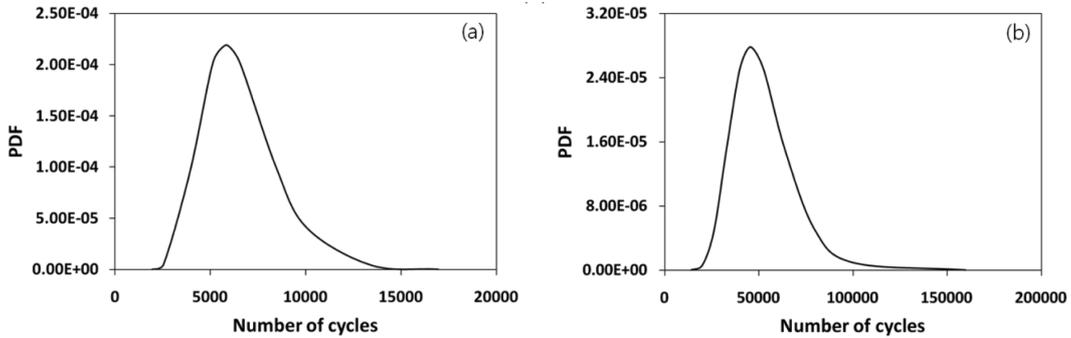


Fig. 6 Lognormal distribution PDF for crack length 11 mm (a) FT-1020, (b) FT-1030

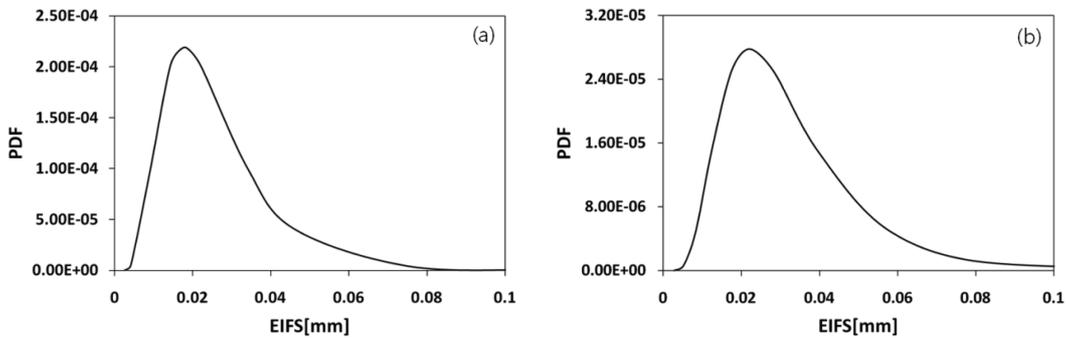


Fig. 7 EIFS distribution PDF (a) FT-1020, (b) FT-1030

failure) of FT-1020 specimen and the fatigue life (50,100 cycles at failure) of FT-1030 specimen and the standard deviation in these distributions is 0.3 from Table 2 for each specimen. The probability density function (PDF) of two specimens at failure is presented in Fig. 6.

The distribution of EIFS is determined by back-extrapolating distribution of fatigue cracks according to Paris crack growth function to zero time or some reference time serving to represent the initial time of the assessment. The distribution of EIFS is expressed in Fig. 7. The average value of EIFS was 0.0267 mm for FT-1020 and 0.0222 mm for FT-1030.

### 3.3 EIFS distribution according to stress level

Several studies to obtain the EIFS using data at different stress levels were carried out. They tested identical specimens at 90, 120 and 160 MPa with the same  $R$  (Koolloos *et al.* 2003). The difference in measured and predicted fatigue life was verified to be sensitive to the stress level used to calculate the EIFS. Therefore, this point has to be considered for more reliable fatigue life prediction. In our study, the values of EIFS were obtained for two stress levels under different  $R$ . To obtain the best prediction of fatigue life, the linear approximation of EIFS was presented in the following equation (Moreira 2005).

$$EIFS \approx a \cdot \sigma + b \quad (4)$$

EIFS is expressed in mm and  $\sigma$  in MPa.  $Y$  axis of (4) is an average value of EIFS distribution and  $X$  axis of (4) is  $\sigma$ .  $\sigma$  is different forward loads by a constant stress for developing detectable flaw distribution. In this study, the constants  $a$  and  $b$  were found to be  $4E-05$  and  $0.0155$ . This study shows the high ability of the EIFS distribution to predict fatigue life for a given structural detail.

#### 4. Multiple crack fatigue analysis considering EIFS distribution

The MSD refers to the accumulation of widespread fatigue cracking in structural details. It can be especially damaging when cracks initiate at several adjacent fastener holes in a mechanically fastened joint (Rohrbaugh *et al.* 1994). To reliably predict the capability of a MSD joint, the influences of widespread fatigue cracking and the initial flaw effects are needed.

MSD problems are typically analyzed by using numerical methods rather than theoretical ones because of the complicated interactions between multiple cracks. The use of numerical methods is still limited by the change of the domain geometry due to the simultaneous growth of multiple cracks. With traditional methods, it is necessary to remesh the computational domain as MSD cracks grow. Another important issue regarding fracture problems is the stochastic nature of the applied load and the initial flaw. It is well known that the crack growth mechanism is influenced by the stress sequence and interaction associated with the arrangement of the load spectra. Given that crack growth is driven by the stress intensity factor (SIF), it is crucial to note the importance of the initial flaw size because SIF increases as flaw size increases (Artley 1989, Tong 2001). In this example, the extended finite element method (XFEM) was used to analyze MSD problems under same EIFS distribution in each crack.

##### 4.1 XFEM methodology

Among those numerical methods, the extended finite element method (XFEM) is a prominent one because it does not need to modify the domain geometry as multiple cracks grow. Besides the continuous field of the conventional FEM, the approximated displacement field is added for discontinuous fields to represent the displacement jumps across cracks. To do so, the displacement field can be enriched by a set of discontinuous functions. The enrichment consists of the step enriched function for nodes belonging to elements cut by the cracks and the base functions of the asymptotic solution of linear elastic fracture mechanics (LEFM) for nodes of elements containing crack tips. The link-up of two cracks is easily implemented in this method (Zi and Belytschko 2003, Zi *et al.* 2004).

Each crack is described implicitly by the signed distance function  $f^c$ . The signed distance function  $f^c(x)$  is defined as the minimum distance from a point  $x$  to a crack  $\Gamma_c$ . Given the nodal values of the signed distance function  $f_i^c$ , the signed distance function in the domain  $\Omega$  is interpolated by means of finite element shape functions (Zi and Belytschko 2003, Zi *et al.* 2004).

$$f^{(c)}(x) = \sum_I N_I(x) f_I^c \quad (5)$$

In addition, the end points of each crack need to be specified to describe the crack geometry. Although other interpolation techniques such as the moving least squares (Belytschko *et al.* 1994) are available, Eq. (5) is the simplest yet that is accurate enough to capture the crack growth. We use

the vector level set method developed by Ventura *et al.* (2002, 2003) where a detailed discussion for updating the level set can be found.

The Paris equation is used for multiple crack growth same as single crack analysis. For mixed mode fracture problems,  $K$  is the equivalent mode I stress intensity factor

$$K = \sqrt{K_I^2 + K_{II}^2} \quad (6)$$

Growing multiple cracks require the crack increments computed for individual cracks for a number of fatigue cycles. We control the size of the crack increment  $\Delta a_{ctl}$  at the crack tip where  $\Delta K$  is the maximum. Once that crack tip is identified, we calculate the corresponding number of fatigue cycles  $\Delta N$  with  $\Delta a_{ctl}$  from Eq. (1). The crack increment  $\Delta a_i$  of crack tip  $i$  is obtained by using the corresponding  $\Delta K_i$  of the crack tip  $i$  and  $\Delta N$ . The direction of the crack growth  $\theta$  is determined based on the maximum hoop stress criterion

$$\theta = 2 \arctan \frac{1}{4} (\rho k \pm \sqrt{\rho k^2 + 8}) \quad (7)$$

Where  $\rho k = K_I/K_{II}$  is the ratio of the mode I stress intensity factor to the mode II one. The stress intensity factors are calculated by interaction integral (Moes *et al.* 1999, Yau 1980, Moran and Shih 1987).

#### 4.2 Experiment of multiple crack specimens

To validate the analysis method, the experimental activity was performed on two kinds of multiple site damage specimens. These specimens are plates with some cracks emanating from four holes as illustrated in Fig. 8. The specimen has five initial crack positions. The all initial crack lengths in Fig. 8 are 1mm. When the model parameters of Paris' law were  $C = 2.5E-10$  ( $\text{MPa}\cdot\text{m}^{0.5}$ ) and  $m = 2.83$ , the XFEM code produced a similar result to the experimental observation as shown in Table 3. Also, the crack propagation results of MSD specimens obtained from the experiment and computational simulation are very similar as shown in Fig. 9. Fig. 10 is the comparison of crack growth length behavior according to load cycles for Paris equation and experiment results.

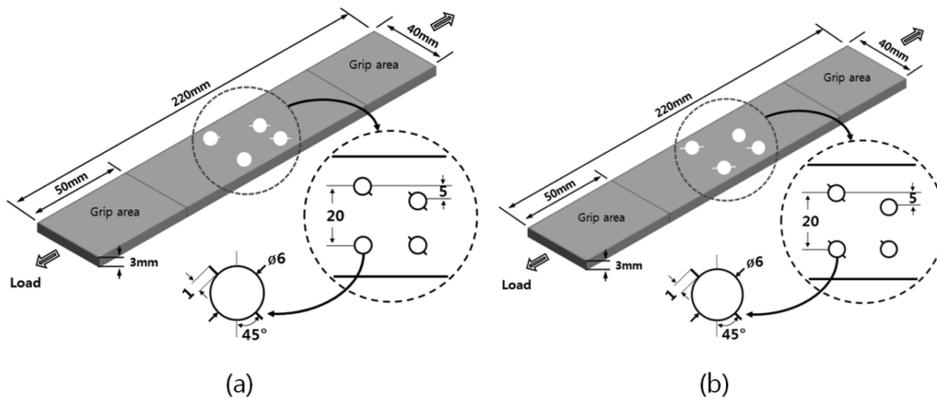


Fig. 8 Details and form of MSD specimens (a) Type 3 (3F01, 3F02), (b) Type 4 (4F01, 4F02)

Table 3 Load condition and load cycles at failure of software and experiment for MSD specimens

Specimen	Force (kN)	Stress Ratio (R)	Number of Cycles at Failure		
			Paris	Test	
Type 3	3F01	3.77~12.58	0.3	42,350	36,353
	3F02	3.77~9.43	0.4	147,000	149,749
Type 4	4F01	3.81~12.68	0.3	45,310	38,970
	4F02	3.81~9.51	0.4	158,000	165,065

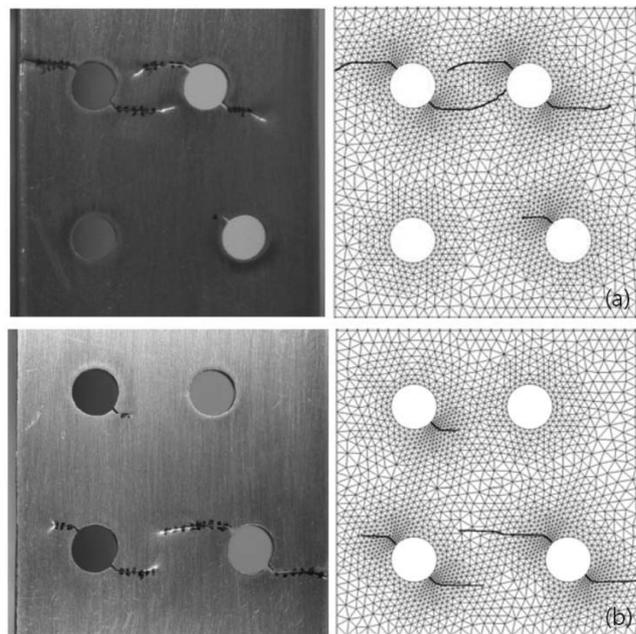


Fig. 9 MSD cracks propagation path for test result and analysis program result (a) Type 3, (b) Type 4

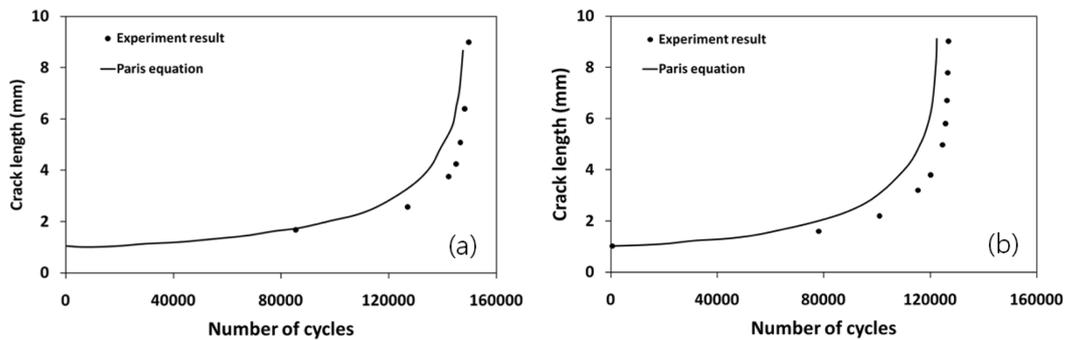


Fig. 10 Crack growth length behavior according to load cycles for Paris equation and experiment results (a) 3F02, (b) 4F02

### 4.3 Example of MSD analysis considering EIFS distribution

To analyze the effect of EIFS distribution in MSD problems, we assume the same EIFS distribution in each initial crack position of 3F01 (Fig. 8(a)) under constant stress. Of course, each initial flaw size distribution and each crack position have different distribution and position.

The EIFS distribution is used from obtained EIFS distribution for FT-1030 (Fig. 7(b)) of the centrally-cracked specimens. Figs. 11(a) and (b) are the EIFS distribution and the EIFS distribution at reference point. The EIFS distribution at reference point is derived for numerical analysis. When we defined each initial crack size according to the EIFS distribution, initial crack sizes in the EIFS distribution are very small to assign crack length on mesh of specimen in our software. Fig. 11(c) is fatigue life cycles of MSD specimen under the same EIFS distribution of each initial crack. As we see in Fig. 11(c), 36,353 cycles for 3F01 experiment at failure is included in the obtained life cycle distribution. If a probabilistic method applies to consider the EIFS distribution in several initial flaws, the more reasonable prediction can be achieved for MSD problems.

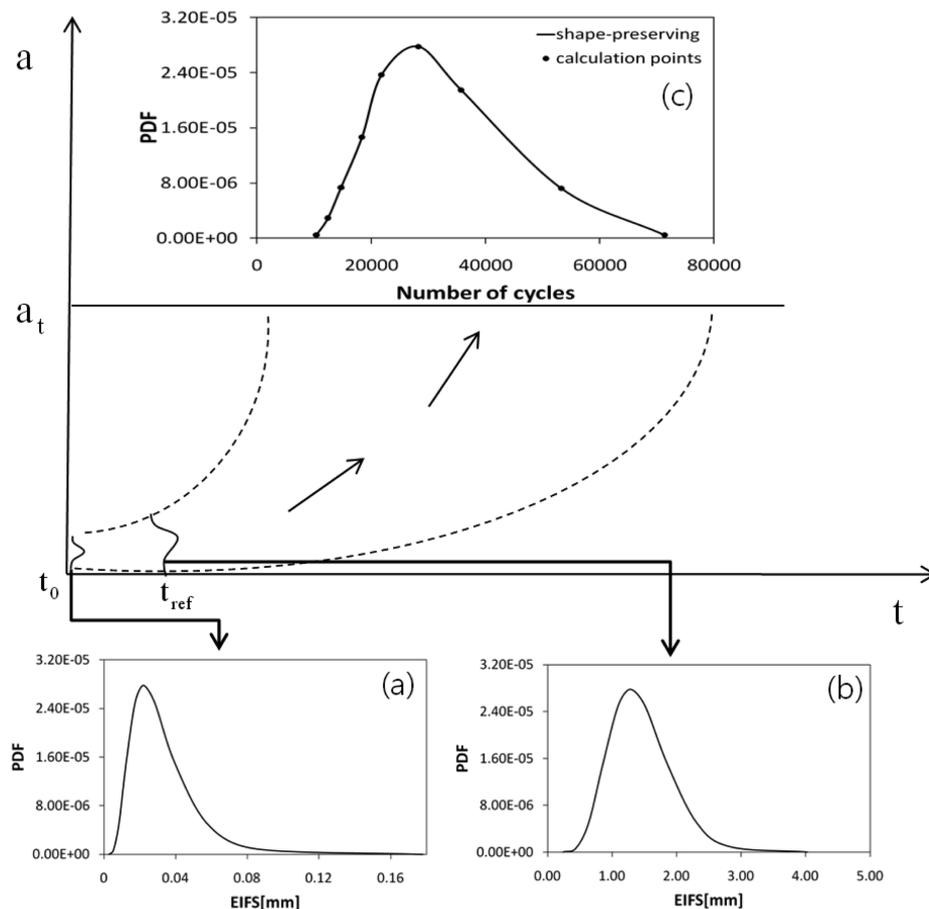


Fig. 11 MSD analysis result of 3F01 specimens (a) EIFS distribution PDF, (b) EIFS distribution PDF in reference time, (c) fatigue life cycle PDF of 3F01 MSD specimen at failure

## 5. Conclusions

Fatigue tests were conducted on the small centrally-cracked specimens that are made of Al 2024-T3. To calculate the value of EIFS, the Paris equation and back-extrapolation technique are used in final crack size distribution obtained from single crack growth length data. The EIFS calculation requires the large amount of test data to develop a reliable EIFS distribution. A distribution of crack lengths at failure can be assumed as a lognormal distribution based on previous researches. From the obtained EIFS distribution, it can determine the initial crack size of structure for crack growth analysis.

Average values of EIFS, at each stress level, were found to follow a dependence on the stress of the type  $EIFS \approx A_1 \cdot \sigma + A_2$ . This relation was identified from several researches as well as this study. More reliable fatigue life can be predicted in this relation between average of EIFS and  $\sigma$  that are different forward loads by a constant stress for developing detectable flaw distribution.

The example problem considering EIFS distribution for MSD was performed by using XFEM. To apply EIFS distribution to MSD specimens, first of all, an experimental activity was performed on two kinds of MSD specimens. The experimental test results were used for the validation of the XFEM. The XFEM code produced a similar result for the experimental observation. Also, crack propagation paths of MSD specimens were nearly the same with experiment result and software result. Although we assumed same EIFS distribution in all crack positions under constant stress, the effect of EIFS distribution was analyzed in MSD specimens.

Based on results of MSD example, several conclusions can be drawn:

- (1) The result was reasonable. The life cycle of experiment result used for MSD is included in the obtained life cycle distribution of the MSD example considering the EIFS distribution.
- (2) The MSD phenomenon can be handled in a logical way because it is one of the possible statistical configurations for various initial flaw cases.
- (3) The damage tolerance design has been the basis of the regulations concerning the prevention of in-service degradation of the structural integrity. The fatigue design of aerospace components is presently approached by using the damage tolerance criterion. The MSD structure considering the EIFS can be used for the damage tolerance design based on life cycle maintenance management.

Our further research will cover probabilistic analysis considering the EIFS distribution under variable loading for MSD. Also the correlation between load spectra properties and MSD crack propagation will be studied.

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